

**The Impact of Corporate and Personal Taxes on the Valuation of Employee
Stock Options**

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Abstract

We incorporate the impact of corporate and personal taxes into the valuation of employee stock options. We provide three new results. First, we show that the fair value of the ESO, defined by the Financial Accounting Standards Board (FASB) in Financial Accounting Standards (FAS) No. 123, Accounting for Stock-Based Compensation, as the price that a willing investor would pay for the security, is less than the value given by a call option pricing model but greater than the value given by a warrant pricing model. Second, we show that the true economic cost to the company is much less than the fair value. Third, we show that the value to the grantee is much less than the fair value, but is greater than the economic cost to the company.

I. Introduction

Employee stock option (ESO) grants are an important form of compensation used by many firms. Of the 1502 firms reported in the 2003 Execucomp database, 86% made option grants. Over 73% granted options to the CEO, but 85% also granted options to non-executive employees. Over 92% of “new economy” companies granted options, but even 86% of “old economy” companies granted options.¹ ESO grants are not restricted to rapidly growing firms, as indicated by 74% of utilities granting options. These data indicate widespread option grants that are not limited to senior executives and that are unique to high-tech industries.

Not only do many companies grant ESOs, but the cumulative option overhang is often quite large. Compustat reports the number of shares reserved for conversion due to employee stock options, convertible bonds, convertible preferred stock, and warrants. While not all of these potentially convertible shares are due to stock options, anecdotal evidence suggests that

¹ Similar to Murphy (2003), we define new economy firms as those in the computer, software, internet, telecommunications, or networking fields, which are firms with the following SIC codes; 3570, 3571, 3572, 3576, 3577, 3661, 3674, 4812, 5045, 5961, 7370, 7371, 7372, or 7373. We define old economy firms as those with an SIC

about 80% of shares reserved for conversion by the typical firm are due to unexercised stock options. Over 600 firms listed on Compustat in 2003 have potentially convertible shares exceeding five percent of the total number of outstanding shares, and over 170 have convertible shares exceeding 15 percent of outstanding shares.² Thus, option grants are an important element in the compensation plans and capital structures for many firms.

Considerable attention is being focused upon ESOs as a result of the Financial Accounting Standards Board's (FASB) proposed amendment to Financial Accounting Standards (FAS) No. 123, Accounting for Stock-Based Compensation, which requires that ESO grants be reported as a compensation expense in a company's financial statements.³ Prior to this amendment, companies could choose to report option related information in the footnotes rather than in the financial statements. Thus, the amendment will provide increased transparency to investors concerning the level of incentive compensation provided by a company. Although the mandated expensing will improve transparency and reduce reported earnings, it will not affect cash flows, because ESOs can be expensed only for shareholder reporting purposes and not for tax purposes.

There is evidence that corporate responses to this forced transparency will be mixed. For example, over 500 companies have recently decided to voluntarily report option expenses on their financial statements, including Ford, General Motors, and Coca-Cola.⁴ On the other hand, some companies have announced plans to modify or eliminate their option grant programs. One notable example is Microsoft, which implemented an option buy-back plan in 2003 and announced its intent to no longer issue options. This is quite a change for a company that had 802 million outstanding options in 2002 worth about \$18 billion dollars.

But changes in behavior are not limited to Microsoft. A recent survey by Mellon Human Resources & Investor Solutions shows that out of 107 responding companies currently granting options, 45% indicate that they will eliminate eligibility for option grants to non-salaried

code less than 4800 and which are not new economy firms. We define utilities as firms with SIC codes between 4900 and 5000.

² These are by no means small firms. For those with an option overhang of at least 5%, the mean sales are over \$330 million, a value that is comparable to the mean sales of \$414 million for firms with fewer convertible shares (the t-statistic for a test of means is only 0.95). For the firms with an overhang of greater than 15%, the mean sales is \$399 million.

³ The effective date is for financial statement reported after June 15, 2005

⁴ See Johnson, C. (2004).

employees and 16% will eliminate eligibility for option grants to non-executive managers.⁵ Interestingly, none are eliminating eligibility for grants to executives, and very few are eliminating eligibility for grants to board members. With respect to plans for issuing grants (versus eligibility for grants), only about 20% of the firms indicated a planned cut in grants to executives, and most of those cutting executive grants indicated that they would “make up” by paying more in cash incentives. For non-executive managers, over 54% indicated that they would reduce option grants, and most indicated that they would not make up the grants with other compensation.

An accurate valuation of ESOs is important for expensing—otherwise the SFAS 123 reporting requirement will obscure rather than clarify ESO costs. An accurate assessment of the economic cost of ESOs is even more important for corporate decision-makers contemplating changes in their incentive programs. Although taxes clearly play an important role in the valuation of other securities and projects, the existing literature does not incorporate the impact of corporate or personal taxation into the valuation of ESOs. We fill this gap in the literature and provide three new and somewhat surprising results: (1) incorporating the impact of corporate taxation results in a “fair value” for an ESO that is slightly higher than when corporate taxes are ignored; (2) the economic cost of an ESO to a corporation’s shareholders is substantially less than the fair value; and (3) the after-personal-tax value to an ESO grantee is substantially less than the fair value, but is greater than the economic cost to the company in most realistic situations. Following is a more detailed discussion of the issues surrounding each of these new results.

SFAS 123 requires that the “fair value” of an ESO be expensed, and defines this as the price a willing investor would pay a willing seller. Ideally, this fair value could be obtained by observing market prices for otherwise identical securities traded in the marketplace. When market prices are not observable, which is typically the case for ESOs, FASB allows the fair value to be estimated using a technique that is “based on established principles of financial economic theory” and that “reflects any and all substantive characteristics of the instrument.”⁶ SFAS 123 specifically mentions several methods for estimating the fair value, including the Black and Scholes (1973) option pricing model and lattice models (including the binomial

⁵ See <http://www.mellon.com/hris/educate/publications/whitepapers.html>.

⁶ SFAS 123, December 2004, Appendix A, section A8, p. 39.

model). In choosing a model, SFAS 123 states that the “selection of a valuation model will depend on the substantive characteristics” of the security and that a “model that is more fully able to capture and better reflects those characteristics is preferable.” Thus, estimation of fair value should incorporate the substantive characteristics of ESOs that distinguish them from standard call options and warrants. Warrants, ESOs, and stock options have substantive differences in their exercise characteristics, cash flows at exercise, tax treatment, dilution effects, and liquidity, and each of these affects the value of an ESO relative to a stock option or warrant. As such, these differences should be modeled when establishing the fair value of the ESO for SFAS 123 purposes.

SFAS 123 specifically mentions two such differences in exercise characteristics: the possibility of forfeiture and that of early exercise. An ESO might be forfeited because the employee leaves the company before the grant is vested or leaves the company after the grant is vested but while the option is out of the money. Early exercise can be caused by two different reasons. First, a grantee is not allowed to sell an ESO, so the only way to raise cash for liquidity purposes, such as payment of college tuition for a child, is to exercise the ESO before its expiration date. Second, option grants often represent a large portion of a grantee’s wealth, causing the grantee’s portfolio to be less diversified than the grantee might desire. In this case, the grantee might exercise the option early and sell the stock to improve diversification. Huddart (1994) states that conversations with compensation program administrators indicate that about 95% of grantees sell their stock on the same day that they exercise the ESO. This monetizing of option wealth confirms that grantees face liquidity needs and/or have desires to diversify. FASB recognizes the likelihood of early exercise and encourages companies to incorporate early exercise into their pricing models. When using the Black-Scholes model to value ESOs, FASB suggests using a shorter “effective” time until expiration than the actual expiration date of the options. Huddart (1994), Kulatilaka and Marcus (1994), Rubinstein (1995), and Hull and White (2004) explicitly model forfeiture and/or early exercise as functions of stock price and the grantee’s utility function within the context of a binomial model.⁷ They find that the resulting

⁷ The impact of the utility function in these models is to cause early exercise for some nodes in the lattice if the utility of the cash from exercise is greater than the utility of holding the risky option. Given these early exercise dates, these models proceed to value the ESO in the same manner as standard binomial models, except that the early exercise dates are used at those nodes (this is analogous to the way a binomial model is used to value American options on dividend paying stock). Although the result is itself dependent upon utility functions, their approach is consistent with the SFAS 123 recommendation that the fair value should include the impact of early exercise.

option value is much less than that provided by the unadjusted Black-Scholes model and that the value is significantly different than the value obtained from the Black-Scholes model using an equivalent effective exercise date.

Another substantive difference between ESOs and call options is that ESOs are like warrants, which generate cash proceeds for the firm and increase the number of outstanding shares if exercised. This impacts valuation because, like a warrant, an ESO's underlying asset is not the stock price, but rather is the firm's quasi-equity, often defined as the combined value of the common stock and ESOs.⁸ Galai and Schneller (1978) modify the Black-Scholes formula to incorporate this characteristic for the pricing of warrants.⁹ The impact of this modification for an individual firm depends upon the degree of overhang. As we show later in the paper, for a company with a 5% overhang, the warrant pricing model produces a fair value that is about 3.2% lower than the Black-Scholes value for a typical grant.¹⁰ The difference between the warrant price and the ESO price becomes larger as the overhang increases.¹¹

Yet another difference between an ESO and an option (or a warrant) is the corporate tax treatment if the ESO is exercised. If a grantee exercises an ESO, then the corporation may deduct from taxable income an amount equal to the difference between the stock price at the time of exercise and the exercise price.¹² The net effect is that the proceeds to the company at the time of exercise are greater than from those of an otherwise identical warrant. In essence, the exercise of the ESO creates a tax shield for the company. This tax shield might affect a corporation's behavior in the four ways that are described below.

⁸ Core, Guay, and Kothari (2001) show the importance of explicitly recognizing the impact of this quasi-equity when estimating the diluted earnings per share.

⁹ The correct volatility is the volatility of the quasi-equity rather than the volatility of the equity. See Crouhy and Galai (1991) and Bensoussan, Crouhy, and Galai (1995) for a discussion of issues associated with estimating this volatility.

¹⁰ Interestingly, SFAS 123 permits, but does not require, the warrant pricing adjustment. They state that in efficient markets, the stock price already reflects the presence of option grants. This is true, but the ESO is a derivative security whose value is contingent upon the value of the quasi-equity (i.e., the combined stock and option value) and not just the stock price. As such, the relevant "price" of the underlying asset is the quasi-equity price and the relevant volatility is the quasi-equity volatility. It is possible to use the stock price as the price of the underlying asset, but this requires the user to explicitly adjust the volatility used in the model throughout the life of the option. When applied correctly, this will give the same answer as the warrant pricing model, but not the same answer as the standard Black-Scholes option pricing model with a constant variance. See Bensoussan, Crouhy, and Galai (1995).

¹¹ Rubinstein (1995) incorporates the impact of exercise proceeds (and the accompanying change in the number of outstanding shares) within the context of a binomial model, but does not isolate its impact on fair value.

¹² This is true for nonqualified options but the situation is different for tax-qualified options (also called incentive options). However, Hall and Liebman (2000) report that about 95% of all option grants are for nonqualified options.

First, if ESOs are a substitute for salary, then firms with high marginal tax rates might prefer to provide compensation in the form of salary (which is immediately deductible) rather than in the form of options (which have a delayed and uncertain deduction). Hall and Liebman (2000), Core and Guay (2001), and Klassen and Mawani (2000) find mild support for this hypothesis.

Second, firms with high marginal tax rates might prefer to issue nonqualified options rather than tax-qualified options, since firms receive a tax deduction when nonqualified options are exercised but not when tax-qualified options are exercised. However, Austin, Gaver, and Gaver (1998) and Madeo and Omer (1994) do not find empirical support for this hypothesis.

Third, the ESO tax shield might affect a firm's marginal tax rate and, hence, its use of debt. Hanlon and Shevlin (2002) examine firms in the Nasdaq 100 and find that their actual tax burden was much lower than the reported tax burden after taking into consideration the ESO deduction. Graham, Lang, and Shackelford (2004) incorporate the ESO tax deductions when they calculate the marginal tax rates for Nasdaq 100 firms and S&P 100 firms. They find that the marginal tax rate is significantly lower for many firms when the ESO tax deduction is included. They also show that firms with lower marginal tax rates (when ESOs are considered) have lower levels of debt. Thus, they conclude that the ESO tax shield is a substitute for debt tax shields. Kahle and Shastri (2005) show that long-term leverage is inversely related to the level of option related tax benefits. They show that when the tax benefits of options fall, firms tend to issue more long-term debt. They also show that firms issuing equity instead of debt tend to have larger tax benefits from options. They, too, conclude that the ESO tax shield acts as a substitute for the debt tax shield.

Fourth, the tax shield from option exercise might have an impact on firm value. Schwartz and Moon (2000, 2001) incorporate the ESO tax deductions when estimating the value of internet firms within the context of Monte Carlo simulations, but they do not separately estimate the impact of the tax shield on value.

Although these papers recognize the existence of ESO taxation, to the best of our knowledge the existing literature does not explicitly incorporate the impact of corporate taxes into the estimation of an ESO's fair value.¹³ For example, Graham (2003) reviews the impact of

¹³ Although Huddart (1994) explicitly recognizes the corporate tax deduction at exercise, he does not appropriately incorporate this into the valuation of an ESO. He defines the after-tax "cost" to the company to be $(S_T - X)(1 - \tau_C)$ if

taxes on corporate finance decisions. Of the 166 papers he cites, only eight examine any of the four aspects of taxes and option-based compensation identified above and none addresses the impact of corporate taxes on an ESO's fair value. Core, Guay, and Larcker (2002) survey 104 papers in the area of equity compensation and incentives, none of which address the impact of taxes on the ESO's fair value. Chance (2004) reviews 113 papers related to the valuation and the expensing of stock options, none of which address the impact of taxes on the ESO's fair value. We fill this gap in the literature in Section III, where we introduce the effect of corporate taxes and provide a closed-form solution for the fair value of an ESO. Our approach is to derive the value of a hypothetical tax-advantaged warrant that allows the company to take the same tax deduction at exercise as if the warrant were an ESO. The value of this hypothetical warrant is the ESO's fair value (i.e., the price that a willing investor would pay for such a security). We denote the value of such a hypothetical warrant at time t as $\omega_{H,t}$. We show that the fair value is less than the value resulting from application of the Black-Scholes option pricing model but that the fair value is slightly higher than value resulting from a standard warrant pricing model.

In Section IV, we derive our second result, that the economic cost of an ESO to a company is substantially less than its fair value. If a company has no outstanding ESOs (or other convertible securities), then the common shareholders have a 100% claim upon the value of the quasi-equity. If ESOs have been granted, however, the stockholders must share the quasi-equity with the grantees, and this dilution of their ownership represents an economic cost. Shareholders presumably incur this cost because the benefits from granting ESOs, perhaps due to a closer alignment of employees' interests with their own, outweigh the costs. In other words, shareholders expect the option grants to cause the total value of the quasi-equity to increase, so that even if they must share the quasi-equity, their resulting "piece of the pie" will be larger.

There are a number of studies that have tried to identify the net impact of this cost-benefit tradeoff, but with mixed results. Rees and Stott (2001) and Bell, Landsman, Miller, and Yeh (2002) conclude that investors perceive option grants to add value, while Aboody (1996) and Aboody, Barth, and Kasznik (2004) conclude that the net impact hurts value.

the ESO is exercised and 0 otherwise, where S_T is the stock price at exercise, X is the exercise cost, and τ_C is the corporate tax rate. He then incorporates this within a binomial model using a lattice based upon the stock price rather than upon the quasi-equity. This expression accurately reflects the cash flows at exercise, but it is not the appropriate "cost," since stock price at exercise must itself reflect the after-tax proceeds. In other words, Huddart treats ESOs as if they were tax-deductible stock options rather than tax-deductible warrants. In addition, Huddart does not separately identify or illustrate the impact that corporate taxes have on the fair value.

Rather than try to determine the net impact of option grants, which includes the economic benefits arising from incentive alignment, we focus on the economic cost of option grants. The value of the tax shield from option exercise serves to augment the total value of quasi-equity, in exactly the same way that the tax shield from interest deductions augments the value of an unlevered firm in the familiar Modigliani and Miller (1963) model. As we show in Section III, the fair value (as perceived by a willing purchaser) already reflects the value of this tax shield. Therefore, the net economic cost to the shareholders is the gain that the shareholders receive from the value of the tax shield reduced by the fair value that they give away to grantees. We show in Section IV that this economic cost is equal to the fair value ($\omega_{H,t}$) reduced by a factor of $(1-\tau_C)$, where τ_C denotes the corporate tax rate. Thus, the economic cost to shareholders is substantially less than the fair value.¹⁴

Yet another distinct “value” of an ESO is the value to the grantee. This may differ from the fair value and the economic cost for at least three reasons. First, ESOs are typically not transferable. Indeed, if employees traded the ESOs, then the ESO would no longer provide its intended incentives. But the inclusion of an ESO in a grantee’s portfolio might reduce diversification to such an extent that the resulting portfolio may not be optimal from the grantee’s perspective. This means grantees may value ESOs less than an investor who is able to diversify, an aspect of ESO valuation that has been addressed in the literature. For example, Huddart (1994) specifies a power utility function for grantees. Using a binomial approach, he estimates the amount of cash a grantee would exchange for the ESO (while maintaining constant utility) and finds that the value of a European ESO is much less than the corresponding Black-Scholes option value (which is an allowable value under SFAS 123). Hall and Murphy (2002) also specify a utility function for grantees. Using a certainty-equivalent approach in a continuous time model, they determine the amount of cash that a grantee would be willing to take in exchange for the ESO while maintaining. They also find that this value is much less than the Black-Scholes value. Meulbroek (2001) assumes that grantees would be willing to hold the market portfolio. She compares the risk-return trade-off for the market portfolio with the risk-return trade-off for an ESO. Using these relationships, she finds the implied discount rate such

¹⁴ Huddart (1994) defines the economic cost to the company as the same amount that a third party would charge the company to defease the ESO by providing to the granting company an amount equal to the company’s after-tax cash flows at expiration. This defeasance value will be different than the economic cost as we define it, since the

that discounting the expected future cash flows of an ESO at this discount rate would produce the same risk-return trade-off as for the market portfolio. She also finds that the resulting value of an ESO to a grantee for a typical NYSE firm is about 70% of the value given by the Black-Scholes model.

The analyses cited above assumed the ESO would be held until expiration. But because ESOs can't be sold, grantees are likely to exercise their ESOs non-optimally, and this early exercise is a second difference between ESOs and options. As we noted earlier, Huddart (1994), Kulatilaka and Marcus (1994), Rubinstein (1995), and Hull and White (2004) explicitly model forfeiture and/or early exercise as functions of stock price and the grantee's utility function within the context of a binomial model. They show that grantees exercise early at many nodes, leading to a value that is substantially lower than the value obtained with a restriction on early exercise.

A third difference between ESOs and options (or most other typical investments) is due to personal taxation, which we address in Section V. As we show in Section V, an ESO is worth less than its fair value as a result of its personal taxation. Specifically, we find that the difference in an ESO's value to a grantee and to a willing investor is *not* due to the tax on the gain at exercise, because grantees and investors must each pay this tax. Instead, it is due to the lost deduction for the initial cost of the instrument. In other words, a willing investor is allowed to deduct the basis (which is the price paid for the instrument, i.e., the fair value as of the grant date) when calculating taxes, but the grantee has a zero-basis for tax purposes. Thus, the difference in value to a grantee versus that to a willing investor is the present value of this lost tax deduction. This lost deduction substantially reduces the value to a grantee.

Prior studies on the value of an ESO to the grantee, which focused on illiquidity and lack of diversification effects, concluded that ESOs are an inefficient form of compensation because the value to the recipient is less than the cost to the company.¹⁵ However, as we show in Section V, the value to the grantee will typically be greater than the economic cost to the company when only the impact of taxes is considered. Thus, ESOs may not be as inefficient a form of compensation as suggested in the literature.

defeasance value will depend upon the tax rate of the defeasing company as well as the tax rate of the company that grants the ESO.

¹⁵ See Huddart (1994) and Hall and Murphy (2002).

The remainder of the paper is organized as follows. Section II identifies the differences between employee stock options and warrants due to corporate and personal taxation, and describes the approach we take to incorporate the impact of taxation on the value of an employee stock option. This approach begins with the standard warrant pricing formula and modifies it to reflect the after-tax cash flows that are unique to an employee stock option. In Section III, we demonstrate that the standard warrant pricing formula already incorporates the impact of personal taxes, even though the tax rate does not explicitly appear in the warrant pricing formula. We then derive a solution for the fair value of an ESO, which is the value of an otherwise identical warrant but one that has the same corporate tax treatment as an employee stock option. In Section IV we show that the economic cost to the company is substantially less than the fair value. In Section V, we incorporate the impact of personal taxes and find the after-personal-tax value to a grantee. Section VI is a brief summary.

II. Overview of the Valuation Process: A Comparison of Employee Stock Options and Warrants

The restriction on trading ESOs precludes the direct use of no-arbitrage methods to price them, because it is impossible to construct and continuously balance an arbitrage portfolio containing ESOs. Thus, the valuation of an ESO must instead invoke the principle that assets with identical cash flows should have identical values, even though it would not be possible to engage in arbitrage if the two assets had different values. We will value an ESO relative to an otherwise identical conventional warrant after making adjustments to incorporate the effect of corporate and personal taxes.

In the next three sections we derive three values for an ESO (the fair value to a willing investor, the economic cost to a company, and the after-personal-tax value to a grantee). Since taxation plays an important role in these derivations, first we show that the price of a regular warrant as calculated using the standard warrant pricing model is independent of the personal tax rate. Intuitively, this is because hedging gains and losses are treated symmetrically for tax purposes in the no-arbitrage portfolio that underlies the resulting warrant pricing formula. However, this does not mean that personal taxes are irrelevant to the investor. Taxes impact the magnitude of arbitrage profits that would prevail if the option were mispriced, taxes impact the

rate of return on the no-arbitrage portfolio, and they impact the payoff to a willing investor who buys the warrant and at expiration either exercises it if it is in the money or lets it expire if it is out of the money. However, the personal tax rate does not affect the hedge portfolio's composition, the resulting no-arbitrage p.d.e., or the boundary condition for the p.d.e. Thus, personal taxes do not affect the pricing of a regular warrant from the perspective of a willing investor. This means that investors who differ only in their personal tax rate will agree on the price of a warrant.

We then use this result to derive the value of a hypothetical tax-advantaged warrant that is identical to a conventional warrant except that the hypothetical warrant receives the same corporate tax treatment as an ESO. Because the corporate tax deduction at exercise increases the corporation's proceeds relative to a regular warrant, the value of this hypothetical tax-advantaged warrant is greater than that of a regular warrant. The value of this hypothetical tax-advantaged warrant is the fair value of an ESO, because it is the price that a willing investor would pay for a security that has the same payoffs as an ESO. We denote the fair value of this hypothetical warrant at time t as $\omega_{H,t}$.

Next, in Section IV, we derive the economic cost to a firm. As noted earlier, the company receives a tax shield if the ESOs are exercised. This tax shield increases the total value of the firm's quasi-equity, in much the same way that the tax shield of debt increases the value of a levered firm. Thus, the economic cost of an ESO to an existing shareholder should reflect the net impact of the increased value of quasi-equity due to the tax shield but after giving away the ESO. Although the fair value of the ESO also incorporates a share of the tax shield, the remaining value of the tax shield belongs to the shareholders, and this reduces the true economic cost to the shareholders of granting the ESO. We show that the value of the tax shield is proportional to the fair value of the ESO, so the value of the tax shield is $\tau_C \omega_{H,t}$, where τ_C denotes the corporate tax rate. We also show that the true economic cost of the ESO is equal to $(1-\tau_C)\omega_{H,t}$.

In Section V, we use the results from the previous sections (i.e., the valuation of a hypothetical tax-advantaged warrant) to determine the after-personal-tax value of an ESO to a grantee. Notice that the ESO differs from the hypothetical warrant only with respect to the ESO's personal tax treatment. At the time of exercise, a warrant investor and an ESO holder must both pay taxes on the profit of the investment. Let S_T denote the stock price at the time of

exercise, let X denote the exercise price, and let $\omega_{H,0}$ denote the original price paid by the investor for the warrant at time zero. The investor is liable for taxes on the profit, defined as $X - S_T - \omega_{H,0}$. Notice that this profit might in fact be a loss, if $X - S_T < \omega_{H,0}$. But if the investor has other taxable investment income, the loss on this investment may be used to reduce the investor's net taxable income. If the warrant expires out of the money, then the investor may use the loss on this investment, which is the original purchase price of $\omega_{H,0}$, to reduce the investor's net taxable income. Thus, if the investor has a marginal personal tax rate of τ_P , then the investor's after-tax cash flow when the warrant is in the money at expiration is $(S_T - X)(1 - \tau_P) + \tau_P (\omega_{H,0})$. If the warrant is out of the money at expiration, the investor's after-tax cash flow is $\tau_P (\omega_{H,0})$.

Now consider the after-tax cash flows to an ESO grantee at expiration. If the ESO is exercised, the grantee must pay taxes on the gain, which is $S_T - X$, leaving an after-tax cash flow of $(S_T - X)(1 - \tau_P)$. If the ESO is out of the money at expiration, then there is no tax liability. In other words, cash flows from the ESO are taxed as if the investor's basis in the ESO is zero. This is consistent with the IRS's concept of symmetry: The company deducts the exercise proceeds from the ESO as if the basis were zero, and the employee is taxed as if the basis were zero. Note also that this does not result in a double deduction for the ESO. Although SFAS 123 now requires that the company expense its ESOs at the time of the grant, this is only for financial reporting; the IRS does not allow a deduction until the option is exercised. As shown in Table 1, the difference in after-tax cash flows for an investor in the hypothetical warrant relative to an option grantee is $\tau_P(\omega_{H,0})$, irrespective of whether the grant is in the money or out of the money.

Table 1: The After-Tax Cash Flows of a Hypothetical Tax-Advantaged Warrant and an Employee Stock Option at Expiration

	After-tax cash flows if:	
	$S_T > X$	$S_T \leq X$
Warrant	$(S_T - X)(1 - \tau) + \tau_P(\omega_{H,0})$	$\tau_P(\omega_{H,0})$
Employee stock option	$(S_T - X)(1 - \tau_P)$	0
Difference (Warrant-ESO)	$\tau_P(\omega_{H,0})$	$\tau_P(\omega_{H,0})$

Thus, an ESO is worth less to the holder than the otherwise identical hypothetical warrant because the ESO's after-tax cash flows are lower in all states of nature. Because this difference is constant in all states, the after-personal-tax value of an ESO to a grantee is equal to the fair value of the hypothetical tax-advantaged warrant less the present value of an after-tax cash flow of $\tau_P(\omega_{H,0})$. We provide this solution in Section V.

III. The Fair Value of an Employee Stock Option

We derive the fair value of an ESO, which is the price that a willing investor would pay for the ESO. We assume that if a willing investor bought the ESO, then the investor would not have restrictions on trading.

Consider a non-dividend paying company with n shares of stock and m warrants outstanding.¹⁶ Each warrant is for a grant of 1 share of stock, has an exercise price of X , and expires at time T . Let S_t denote the stock price per share at time t and let $\omega_{H,t}$ denote the value of a hypothetical tax-advantaged warrant that permits the company to take the same tax deduction at exercise as with an ESO. As we show later in this section, the tax shield created by the exercise deduction adds value to the firm. Let K_t denote the total value of the exercise tax shield at time t ; let k_t denote the value of the tax shield per ESO and be defined as K_t/m . The total value of equity, denoted by E_t , is nS_t ; the total value of the outstanding hypothetical tax advantaged warrants, denoted by W_t , is $m\omega_{H,t}$. Let V_t denote the underlying value of the firm's

quasi-equity, which we define as the total value of equity plus the total value of the hypothetical warrants minus the total value of the exercise tax shield. Let v_t denote the per share value of V_t ; i.e., $v_t = V_t/n$ (notice that if a firm had no warrants, v_t would simply be the per share stock price). We assume that v_t follows geometric Brownian motion with an instantaneous standard deviation of σ .

As explained in Section III, our first step is to calculate the value that a willing investor would pay for the hypothetical tax-advantaged option, which incorporates the corporate tax deduction at exercise. In Section V we will show the after-personal-tax value of an ESO to a grantee, but now we establish that price a willing investor would pay for a tax advantaged warrant is independent of the investor's personal tax rate.

Lemma: The absence of arbitrage requires that the price a willing investor would pay for a call option, a warrant, or a hypothetical tax-advantaged warrant must be independent of the investor's personal tax rate.

Proof: See Appendix.

As we show in the Appendix, the price of a call option, warrant, or tax-advantaged warrant (based upon solution to a no-arbitrage p.d.e.) is independent of the personal tax rate. Intuitively, this is because hedging gains and losses are treated symmetrically for tax purposes in the no-arbitrage portfolio that underlies the resulting pricing formula. Thus, personal taxes do not affect the pricing of a call option, warrant, or tax-advantaged warrant from the perspective of a willing investor.

Now consider the payoff at exercise from a hypothetical warrant. The hypothetical warrant, denoted by $\omega_{H,t}$, has a payoff at exercise of:

$$\text{Max}[S_T - X, 0] \tag{1}$$

The company may deduct the exercise value of the warrant from income when it is exercised, which provides a tax shield. The stock price after exercise will reflect the proceeds from exercise and the value of any tax savings. The total value of the company at exercise is the company's pre-exercise value, nv_T , plus the proceeds from exercise, mX , plus the tax savings, $m(S_T - X)\tau_C$. The number of shares after exercise is $n + m$. Therefore, the stock price after

¹⁶ For clarity of exposition, we consider only non-dividend paying companies. The impact of dividends does not change our basic results.

exercise is:

$$S_T = \frac{nv_T + mX + m(S_T - X)\tau_C}{n + m} \quad (2)$$

Rewriting (2),

$$\begin{aligned} S_T - X &= \frac{nv_T + mX + m(S_T - X)\tau_C - nX - mX}{n + m} \\ &= \left(\frac{n}{n + m(1 - \tau_C)} \right) (v_T - X) \end{aligned} \quad (3)$$

Substituting (2) into (1) yields the boundary condition at expiration, time T:

$$\omega_{H,T} = \text{Max} \left[\left(\frac{n}{n + m(1 - \tau_C)} \right) (v_T - X), 0 \right] \quad (4)$$

Simplifying yields:

$$\omega_{H,T} = \left(\frac{n}{n + m(1 - \tau_C)} \right) \text{Max} [(v_T - X), 0] \quad (5)$$

Notice that the expression in (5) is the exercise value for

$$\left(\frac{n}{n + m(1 - \tau_C)} \right) \quad (6)$$

call options written on v . Let ω^* denote the Black-Scholes solution for a single one of these options:

$$\omega^* = v_t N(d_1) - X e^{-r(T-t)} N(d_2) \quad (7)$$

$$d_1 = \frac{\ln(v_t/X) + (r + \sigma^2/2)(T-t)}{\sigma \sqrt{T-t}}, \quad (8)$$

$$d_2 = d_1 - \sigma(T-t)^{1/2}, \quad (9)$$

where t denotes the current date and $N(\cdot)$ is the cumulative normal distribution function.

Thus, the value of the hypothetical warrant is equal to:

$$\omega_{H,t} = \left(\frac{n}{n + m(1 - \tau_C)} \right) \omega^* \quad (10)$$

Equations (7) – (10) require as an input an estimate of v_t , the per share value of the underlying asset at time t , but only the stock price is observable at time t . Recall that V_t , the total value of the underlying asset, is the value of an otherwise identical firm that does not have the tax corporate tax shield due to exercise of the ESO. The observed stock price, S_t , and the estimated hypothetical warrant price, $\omega_{H,t}$, both reflect the value of the tax shield. Therefore, the total value of the underlying asset is equal to the total value of equity plus the total value of the hypothetical warrants, less the total value of the tax shield:

$$n v_t = n S_t + m \omega_{H,t} - m k_t . \quad (11)$$

To find an estimate of v_t , we will derive expressions for the value of the tax shield in terms of v_t , and then solve equation (11) for v_t .

Like the hypothetical warrant, the value of the tax shield is also a derivative security that depends upon the value of the underlying asset. The payoff to k at expiration is:

$$\tau_C \text{Max}[S_T - X, 0] \quad (12)$$

Notice that the payoff to k is simply the payoff to the hypothetical tax-advantaged warrant multiplied by τ_C . Therefore, the value of k at time t is:

$$k_t = \tau_C \omega_{H,t} = \tau_C \left(\frac{n}{n + m(1 - \tau_C)} \right) \omega^* \quad (13)$$

We substitute equation (13) into equation (11) and solve for v_t . The value of the firm without the tax shield, v_t , is the stock value plus the warrant value less the tax shield value:

$$n v_t = n S_t + m \omega_{H,t} - m \tau_C \omega_{H,t} \quad (14)$$

Solving for v_t and collecting terms gives:

$$v_t = S_t + (1 - \tau_C) m \omega_{H,t} / n \quad (15)$$

Notice that the value of the hypothetical warrant in equation (10) is an implicit function through equations (7) through (10) and (15).

Notice that if the corporate tax rate is zero, then the solution for the price of the hypothetical tax-advantaged warrant reduces to that of a regular warrant. The corporate tax rates serves to increase the value of the hypothetical tax-advantaged warrant relative to a regular warrant in two ways. First, the presence of the corporate tax rate in the denominator of equation

(11) increases the value of the hypothetical warrant, all else held equal. Second, the presence of the tax rate in equation (15) means that the hypothetical warrant will be exercised in more states of nature than a regular warrant, which again serves to increase the value of the hypothetical warrant relative to a regular warrant. Therefore, the net effect of the corporate tax deduction at exercise is to increase the price of the hypothetical warrant relative to that of a regular warrant.

Thus, the fair value of an ESO is greater than the price given by the standard warrant pricing model. In Section VI we provide fair values for a range of inputs.

IV. The Economic Cost to Shareholders Due to an Employee Stock Option

Let ω_C denote the economic cost to the shareholder per ESO that is granted. The economic cost is the reduction in wealth experienced by the stockholders due to the grant of the ESO, all else held equal. By holding all else equal, we mean that the value of v_t , the underlying asset, reflects any non-tax benefits that would have accrued from issuing ESOs such as improved incentives, increased productivity, and the like; later in this Section we provide an alternative derivation that does not require this assumption. If there were no ESOs (but v_t reflects all benefits of ESOs), then the per share value of the stock would be v_t . With ESOs, the value of the stock is S_t , as defined in equation (15). Thus, the economic cost to the shareholders per share of stock is the change in value of the stock, which is $v_t - S_t$. The economic cost per ESO is $n(v_t - S_t)/m$. Let $\omega_{C,t}$ denote this economic cost. We solve equation (15) for $n(v_t - S_t)/m$, which yields:

$$\omega_{C,t} = \frac{n(v_t - S_t)}{m} = (1 - \tau_C) \omega_{H,t} \quad (16)$$

In other words, the after-tax economic cost to the shareholders of issuing an ESO is $(1 - \tau_C)$ multiplied by the fair value of the ESO. Interestingly, this is true even though the fair value is not deductible for tax purposes at the time it is granted.

Following is an alternative derivation of the company's economic cost of granting an ESO. The "gross" cost to the company is the fair value of the ESO. However, the company now

owns the value of the tax shield. Therefore, the net economic cost to the company (per ESO) is the fair value of the ESO less the value of the tax shield per ESO:¹⁷

$$\omega_{C,t} = \omega_{H,t} - k_t \quad (17)$$

Substituting equation (13) into (17) yields the following expression, which is the same as equation (16):

$$\omega_{C,t} = (1 - \tau_C) \omega_{H,t} \quad (18)$$

Thus, the economic cost to the company is less than the value of the hypothetical warrant because the company receives the tax shield.

How does the economic cost to the company compare with the values when ESOs are expensed in accordance with SFAS 123? Assume that the company correctly values the ESO as the fair value as defined in this paper. Now suppose that the ESO vested immediately at the time of the grant and that the company could actually deduct the cost of the grant at the time it is made. In this case, the change in reported net income would be equal to the economic cost of the grant.

V. The After-Personal-Tax Value of an Employee Stock Option to the Grantee

Section II showed that the net after-personal-tax cash flow of an ESO to a grantee is lower than that of the hypothetical warrant at expiration in all states of nature by the amount $\tau_P(\omega_{H,t})$, where $\omega_{H,t}$ is the initial value of the hypothetical warrant at the grant date. This difference is because the ESO grantee must use zero as the tax basis irrespective of the value of the ESO at grant, while a warrant purchaser will use the warrant's value at purchase as the tax basis. The zero-basis for the ESO increases the grantee's taxes due at expiration vis-à-vis those of the investor who purchased the hypothetical warrant.

An ESO grantee could replicate the cash flows of an ESO by purchasing a hypothetical warrant and borrowing the present value of an after-tax payment of $\tau_P(\omega_{H,t})$ due at time T.

¹⁷ Keep in mind that the fair value already reflects a portion of the tax shield that belongs to the grantees as a result of their claim on value. Thus, netting out the entire value of the tax shield results in the net economic cost to the shareholders. In other words, the economic cost could be expressed as the value of a regular warrant granted to employees plus the grantees' claim on the tax shield minus the total value of the tax shield.

Because this payment is due in all states of nature, the appropriate rate is the risk-free interest rate. We assume that the grantee would pay taxes continuously, so the amount needed today (time t) to fund an after-tax payment of $\tau_P(\omega_{H,t})$ at time T is $\tau_P \omega_{H,t} e^{-r(1-\tau_P)(T-t)}$.¹⁸ In other words, if the grantee borrowed $\tau_P \omega_{H,t} e^{-r(1-\tau_P)(T-t)}$, invested it at the interest rate r , and paid taxes from the investment income, then the grantee would have exactly $\tau_P(\omega_{H,t})$ at time T .

Thus, the after-personal-tax value of an ESO to the grantee, denoted by $\omega_{H,t}$, is the value of the hypothetical warrant less the present value of the lost basis:

$$\begin{aligned}\omega_{E,t} &= \omega_{H,t} - \tau_P \omega_{H,t} e^{-r(1-\tau_P)(T-t)} \\ &= \omega_{H,t} \left(1 - \tau_P e^{-r(1-\tau_P)(T-t)}\right)\end{aligned}\tag{22}$$

VI. Discussion

The previous sections demonstrate that the corporate tax treatment of ESOs provides additional value to the firm in the form of a tax shield if the ESO is exercised. Because this tax treatment increases the net after-tax proceeds to the firm at the time the ESO is exercised, the fair value that a willing investor would pay for an ESO is greater than the value of a regular warrant with the same terms. The additional tax shield at exercise also serves to increase the value of a firm's stock relative to the value it would have if it had granted a regular warrant instead of an ESO. Thus, the economic cost to the firm is substantially lower than the fair value. An ESO grantee does not receive the right to deduct the initial value (i.e., the basis) of the ESO at expiration for tax purposes, whereas a willing investor purchasing a warrant does have this right. This causes the after-personal-tax value of an ESO to a grantee to be substantially less than the fair value.

Following is a numerical example illustrating these three values, summarized in Table 2. We assume that the observed stock price is \$25.00 and that the company has 200 million shares of stock outstanding. The company has granted 10 million ESOs with an exercise price of \$25.00 and ten years until expiration. We assume that the risk-free rate is 5%, and that the standard deviation of the underlying asset is 0.36.

First, suppose that the company uses the standard Black-Scholes option pricing model for a call option, not a warrant. In this case, the estimated value is \$14.29. Now suppose the

¹⁸ This assumes a flat term structure, which is consistent with the assumptions underlying the warrant pricing model.

company uses the standard warrant pricing model, but ignores taxes. This produces a value of \$13.61. This is less than the call option value because the call option does not include the dilutive impact of the additional shares of stock that result when a warrant is exercised.

Assuming a corporate tax rate of 40%, the fair value (i.e., the value of a hypothetical tax-advantaged warrant) is \$14.22.

The value of the tax shield of exercise is \$5.69 per warrant. Therefore, the economic cost to the company from granting the ESO is \$8.53.

Assume that the grantee is subject to personal taxes at a rate of 35%. The after-personal-tax value to the grantee is \$10.62. Notice that the economic cost to the shareholders and the after-personal-tax value of the ESO to the grantees are not equal. Not only is the grant not inefficient, it is super-efficient. This difference in value is not due just to the difference in the corporate tax rate and the personal tax rate.

Stock price	\$25.00
Number of shares of stock	200
Number of ESOs	10
Exercise price	\$25.00
Risk-free rate	5.00%
Maturity (years)	10
Standard deviation of v	0.360
Black-Scholes call option based on S	\$14.29
Standard warrant based on S	\$13.61
Hypothetical warrant	\$14.22
Value of tax shield per warrant	\$5.69
Economic cost to company	\$8.53
Value to ESO grantee	\$10.62

In summary, ignoring corporate and personal taxes leads to substantial biases when considering the cost and value of an ESO.

Appendix: Warrant Pricing and Personal Taxes

Lemma: The absence of arbitrage requires that the price a willing investor would pay for a call option, a warrant, or a hypothetical tax-advantaged warrant must be independent of the investor's personal tax rate.

Following is the proof for this lemma. We begin with a proof for a warrant, and then extend it for a call option and for a hypothetical tax-advantaged warrant.

Consider a non-dividend paying company with n shares of stock and m warrants outstanding. Each warrant is for a grant of γ shares of stock, has an exercise price of X , and expires at time T . Let S_t denote the stock price per share at time t and let ω_t denote the value of a single warrant at time t . The total value of equity at time t , denoted by E_t , is nS_t ; the total value of the outstanding warrants, denoted by W_t , is $m\omega_t$. Let V_t denote the underlying value of the firm, based upon its ability to generate cash flows for investors. Because the only claims on this value are from stockholders and warrant holders, V_t is also equal to the combined value of the equity and warrants issued by a firm, so that $V_t = E_t + W_t$. Let v_t denote the per share value of V_t ; i.e., $v_t = V_t/n$ (notice that if a firm had no warrants, v_t would simply be the per share stock price). Assume that

$$dv = \mu v dt + \sigma v dz \tag{A-1}$$

where μ and σ are constants and dz is a Gauss-Wiener variable.

We begin first by deriving the fundamental partial differential equation (pde) for a warrant, which is a function of the company's value, V . As we show below, the fundamental pde is invariant to the presence of personal taxes. Although this derivation is familiar, we provide it to emphasize its invariance with respect to taxes.

Consider a hedge portfolio consisting of one share of stock and N_ω warrants. The value of the portfolio, H , is:

$$H = S + \omega N_\omega \tag{A-2}$$

Notice that since $V = nv = E + W = nS + m\omega$,

$$S = v - \frac{m}{n} \omega \quad (\text{A-3})$$

Therefore, the value of the hedge portfolio is:

$$H = v - \frac{m}{n} \omega + \omega N_\omega = v + \left(N_\omega - \frac{m}{n} \right) \omega \quad (\text{A-4})$$

Incremental changes in the value of the portfolio are denoted by dV_H .

$$dH = dv + \left(N_\omega - \frac{m}{n} \right) d\omega \quad (\text{A-5})$$

Assume that the warrant price is a function of v (the underlying per share value of the asset) and time:

$$\omega = \omega(v, t) \quad (\text{A-6})$$

$$d\omega = \frac{\partial \omega}{\partial v} dv + \frac{\partial \omega}{\partial t} dt + \frac{1}{2} v^2 \sigma^2 \frac{\partial^2 \omega}{\partial v^2} dt \quad (\text{A-7})$$

Let

$$N_\omega = \frac{m}{n} - 1 / \left(\frac{\partial \omega}{\partial v} \right) \quad (\text{A-8})$$

Substituting (A-7) and (A-8) into (A-5):

$$dH = dv + \left(\frac{m}{n} - 1 / \left(\frac{\partial \omega}{\partial v} \right) - \frac{m}{n} \right) \left(\frac{\partial \omega}{\partial v} dv + \frac{\partial \omega}{\partial t} dt + \frac{1}{2} v^2 \sigma^2 \frac{\partial^2 \omega}{\partial v^2} dt \right) \quad (\text{A-9})$$

Simplifying:

$$dH = - \left(1 / \frac{\partial \omega}{\partial v} \right) \frac{\partial \omega}{\partial t} dt - \left(1 / \frac{\partial \omega}{\partial v} \right) \frac{1}{2} v^2 \sigma^2 \frac{\partial^2 \omega}{\partial v^2} dt \quad (\text{A-10})$$

The hedge portfolio in (A-4) is riskless; therefore, the after-tax return on the portfolio must equal the risk-free rate of return (denoted by r) on an after-tax basis, where τ_p is the personal tax rate on the hedger:

$$\frac{dH}{H}(1 - \tau_p) = (1 - \tau_p)r dt \quad (\text{A-11})$$

Here we assume that the tax rate on hedging returns (which consist of net short term capital gains) is equal to the tax rate on interest income, and that these taxes are paid continuously.

Substituting (A-4), (A-8), and (A-10) into (A-11) yields

$$\frac{-\left(1/\frac{\partial\omega}{\partial v}\right)\frac{\partial\omega}{\partial t}dt - \left(1/\frac{\partial\omega}{\partial v}\right)\frac{1}{2}v^2\sigma^2\frac{\partial^2\omega}{\partial v^2}dt}{v + \left(\frac{m}{n} - 1/\left(\frac{\partial\omega}{\partial v}\right) - \frac{m}{n}\right)\omega} = rdt \quad (\text{A-12})$$

Simplifying yields the familiar partial differential equation governing the value of a derivative security:

$$\begin{aligned} -\left(1/\frac{\partial\omega}{\partial v}\right)\frac{\partial\omega}{\partial t}dt - \left(1/\frac{\partial\omega}{\partial v}\right)\frac{1}{2}v^2\sigma^2\frac{\partial^2\omega}{\partial v^2}dt &= vr dt + \left(-1/\left(\frac{\partial\omega}{\partial v}\right)\right)\omega r dt \\ rv\frac{\partial\omega}{\partial v} - \omega r + \frac{\partial\omega}{\partial t} + \frac{1}{2}v^2\sigma^2\frac{\partial^2\omega}{\partial v^2} &= 0 \end{aligned} \quad (\text{A-12})$$

Notice that (A-12) is identical to the pde for a call option on a stock, except that terms involving the stock price have been replaced with terms involving v , the total per share value of the combined equity and warrants.¹⁹ Notice also that the tax rate of the marginal investor does not appear in (A-12).

We repeat the familiar derivation of the value of a conventional warrant (see, for example, Hull 2004), to highlight the reason why the personal tax rate does not appear in the warrant pricing formula. Letting S_T denote the stock price at expiration, the value of the warrant at expiration, ω_T , must be:

¹⁹ In fact, if the firm has no warrants and we redefine ω to be the stock price per share, then equation (A-12) is exactly the familiar Black-Scholes pde for a call option.

$$\omega_T = \gamma \text{Max}[S_T - X, 0] \quad (\text{A-13})$$

Notice that if this condition is not true, then there would be an arbitrage opportunity. This profit would be taxable, but because the tax rate is less than 100%, the arbitrageur would still have a positive net after-tax profit if (A-13) were not true. Market forces will eliminate the opportunity for such an after-tax profit, so equation (A-13) must hold. Notice also that if the warrant is purchased for the value given by equation (A-13), then there will be no tax liability for someone who purchases the warrant at the price of ω_T at the moment prior to expiration. Thus, the potential presence of arbitrageurs ensures that equation (A-13) must be the appropriate boundary condition for a warrant, even though the equation does not explicitly include personal taxes. This is not to say that a warrant investor will not pay taxes at exercise, but it does say that the boundary condition for ω_T at exercise does not explicitly include taxes. We will have more to say about this later in this section when we provide the equivalent valuation using the binomial approach.

If the warrant is in the money at expiration, the stock price after exercise will reflect the proceeds from exercise. The total value of the company at exercise is the company's pre-exercise value, nv , plus the proceeds from exercise, $m\gamma X$. The number of shares after exercise is $n + m\gamma$. Therefore, the stock price after exercise is:

$$S_T = \frac{nv + m\gamma X}{n + m\gamma} \quad (\text{A-14})$$

Rewriting (A-14),

$$\begin{aligned} S_T - X &= \frac{nv + m\gamma X - nX - m\gamma X}{n + m\gamma} \\ &= \left(\frac{n}{n + m\gamma} \right) (v - X) \end{aligned} \quad (\text{A-15})$$

Substituting (A-15) into (A-13) yields the exercise value boundary condition at expiration, time T:

$$\omega(v, T) = \gamma \text{Max} \left[\left(\frac{n}{n + m\gamma} \right) (v - X), 0 \right] \quad (\text{A-16})$$

Simplifying yields:

$$\omega(v, T) = \left(\frac{n\gamma}{n + m\gamma} \right) \text{Max}[(v - X), 0] \quad (\text{A-17})$$

Notice that the expression in (17) is the exercise value for

$$\left(\frac{n\gamma}{n + m\gamma} \right) \quad (\text{A-18})$$

options written on v . Let ω^* denote the Black-Scholes solution for one such option on v :

$$\omega^* = v N(d_1) - Xe^{-r(T-t)} N(d_2) \quad (\text{A-19})$$

$$d_1 = \frac{\ln(v_0/X) + (r + \sigma^2/2)(T-t)}{\sigma \sqrt{T-t}}, \quad (\text{A-20})$$

$$d_2 = d_1 - \sigma(T-t)^{1/2}, \quad (\text{A-21})$$

and v_t is the per share value of the total company—which is the stock plus the options:

$$v_t = S_t + m\omega/n \quad (\text{A-22})$$

where t is the current date and $N(x)$ is the cumulative normal distribution function. Note that ω^* is an implicit function through equations (A-19) through (A-22).

Thus, the value of the warrant at time t is equal to the adjusted Black-Scholes value:²⁰

$$\omega_t = \left(\frac{n\gamma}{n + m\gamma} \right) \omega^* \quad (\text{A-23})$$

This illustrates that the valuation formula of a warrant does not include the personal tax rate, even if the individual is liable for taxes. The intuition is that the arbitrage condition that drives the price to correspond to Equation (A-23) is not eliminated by taxes. Certainly, taxes reduce the magnitude of the profits that will be available to the arbitrageur if the boundary condition Equation (A-13) is violated, but the potential after-tax profits will still drive the boundary condition to be satisfied. And taxes reduce the risk free return earned on both the risk free investment and the synthetic risk free investment in Equation (A-11), but the tax rate is the same on either portfolio, so whenever the pre-tax returns are equal, then the after-tax returns will be equal. Note also that this equality doesn't depend on the concept of the "marginal investor's"

tax rate. The tax rate may vary from person to person as long as for each person the tax rate on the risk free investment is equal to the tax rate on the synthetic risk free investment.

If we set γ and λ to zero, then (A-23) reduces to the familiar Black-Scholes price for a call option. Thus, call option prices are independent of personal taxes. If we replace the stock price in (A-14) with the after-corporate-tax proceeds for a tax-advantaged warrant (i.e., a warrant that allows the issuing company to deduct the difference between the stock price and the exercise price if the stock is exercised, as described in equation (15) in the body of the paper), then the boundary condition and resulting price are still independent of the personal tax rate. This completes the proof of the lemma.

Because the binomial method is often used for ESO valuations, we demonstrate that the value of any asset with gains and losses that are taxed each period is also independent of the personal tax rate.

Consider a stock price with an initial value of S . At the end of the period, the price will be either S^+ or S^- . There exists a risk-free asset with an initial price of $\$1$. At the end of the period, it will produce a value of R . The end-of-period values for the stock and risk-free asset are pre-tax values and do not reflect the impact of personal taxes on any gains or losses.

Let D_B^+ be the discount factor for a before-tax cash flow in the up state. In other words, D_B^+ is the price of an Arrow-Debreu security that pays $\$1$ in the up state and nothing in the down state. Similarly, D_B^- is the discount factor for a before-tax cash flow in the down state. It is easy to show that

$$D_B^+ = \frac{RS - S^-}{R(S^+ - S^-)} \quad (\text{A-24})$$

and that

$$D_B^- = \frac{-RS + S^+}{R(S^+ - S^-)} \quad (\text{A-25})$$

Now consider any asset that pays A^+ in the up state and A^- in the down state. The current value of such an asset, denoted by A , is

$$A = (A^+ D_B^+) + (A^- D_B^-) \quad (\text{A-26})$$

²⁰ It is easy to incorporate a continuous dividend rate q , paid as a proportion of v in Equation (1). Simply replace v

Substituting equation (A-25) into equation (A-26) yields:

$$A = A^+ \left(\frac{RS - S^-}{R(S^+ - S^-)} \right) + A^- \left(\frac{-RS + S^+}{R(S^+ - S^-)} \right) \quad (\text{A-27})$$

As we show below, this is the same value that results when the analysis is repeated using after-tax cash flows.

Now consider the after-tax cash flows from the stock and the risk-free asset to an investor with a personal tax rate of τ_p . In an up state, the after-tax cash flow of the stock is $S^+ - (S^+ - S)\tau_p = S^+(1 - \tau_p) + (\tau_p S)$. In a down state, the after-tax cash flow of the stock is $S^-(1 - \tau_p) + (\tau_p S)$. For the risk-free security, the after-tax payoff is $R(1 - \tau_p) + \tau_p$. It is easy to show that the after-tax discount rate for the up state, D_A^+ , is:

$$D_A^+ = \frac{RS - S^-}{(R(1 - \tau_p) + \tau_p)(S^+ - S^-)} \quad (\text{A-28})$$

The after-tax discount rate for the down state, D_A^- , is:

$$D_A^- = \frac{-RS + S^+}{(R(1 - \tau_p) + \tau_p)(S^+ - S^-)} \quad (\text{A-29})$$

The after-tax cash flows for asset A are $A^+(1 - \tau_p) + (\tau_p A)$ in the up state and $A^-(1 - \tau_p) + (\tau_p A)$ in the down state. The value of the asset is equal to the after-tax cash flows when discounted at the after-tax discount rates:

$$A = [A^+(1 - \tau_p) + (\tau_p A)] (D_A^+) + [A^-(1 - \tau_p) + (\tau_p A)] (D_A^-) \quad (\text{A-30})$$

Substituting equations (A-28) and (A-29) into (A-30) results in the same value for A as in equation (A-27). In other words, the value of the asset can be found by discounting pre-tax cash flows at the pre-tax discount rates or by discounting after-tax cash flows at the after-tax discount rates.

Just like in the continuous time version, the reason the pricing function is independent of the tax rate is because under our assumptions, the no-arbitrage condition is independent of the tax rate. The reason for this independence is that first, the tax rate in the up state is equal to the

with $v^* = ve^{-r(1-\tau_p)(T-t)}$ in Equations (A-19) through (A-22).

tax rate in the down state which is equal to the tax rate on the risk free asset. Second, the tax basis for the investment is equal to the value of the investment. To see this, recall that the Arrow-Debreu prices are constructed to prevent arbitrage. Suppose you were to form a risk free portfolio (either before-tax or after-tax) consisting of 1 share of S and δ shares of A. Using before-tax cash flows this portfolio would have to satisfy

$$S^+ + \delta A^+ = S^- + \delta A^-$$

so that

$$\delta = -(S^+ - S^-)/(A^+ - A^-).$$

Using after-tax cash flows, the portfolio would have to satisfy

$$\begin{aligned} & S^+ (1 - \tau_P^+) + S_{\text{basis}}^+ \tau_P^+ + \delta(A^+ (1 - \tau_P^+) + A_{\text{basis}}^+ \tau_P^+) \\ &= S^- (1 - \tau_P^-) + S_{\text{basis}}^- \tau_P^- + \delta(A^- (1 - \tau_P^-) + A_{\text{basis}}^- \tau_P^-) \end{aligned}$$

As long as the tax rate in the up state, τ_P^+ , is equal to the tax rate in the down state, τ_P^- , and the basis for tax purposes in the up state, A_{basis}^+ , is equal the basis for tax purposes in the down state, A_{basis}^- , the expression above simplifies to $\delta = -(S^+ - S^-)/(A^+ - A^-)$, which doesn't depend on taxes. In other words, provided the taxes are symmetric for the up and down states, the construction of the hedge portfolio is independent of taxes and the no-arbitrage price of the assets therefore doesn't depend on taxes. Notice that this result does not require that the tax basis be the original cost of the asset, only that it be the same in either state of the world. Also, if the tax rates were different in the up and down states, such as if gains were taxed immediately, but losses had to be carried forward, then the tax rates would enter into the pricing function.

This same general result applies to the warrant. It is easy to derive the value of the warrant in either continuous time or in a binomial approach based upon the after-tax cash flows of the warrant and the after-tax cash flows of the stock and the risk-free asset, and this would provide the same answer as the warrant pricing model in equation (23), which is based upon the pre-tax cash flows of the warrant, the stock and the risk-free asset.²¹

²¹ This assumes that all gains and losses are recognized continuously for tax purposes.

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